In the Footsteps of Matteo Ricci PCTM 2008

Robert McGee, Professor Emeritus, Cabrini College

Carol Serotta, Cabrini College

Kathleen Acker, Ph.D.

The Holy Cross Study Tour of China
"In the Footsteps of Matteo Ricci"

Matteo Ricci

At the end of about 18 pages of discussion of the history of Chinese mathematics , Victor Katz writes:

> *Finally, in the late sixteenth century, with the arrival of the Jesuit priest Matteo Ricci (1552-1610), Western mathematics entered China and the indigenous tradition began to disappear.*

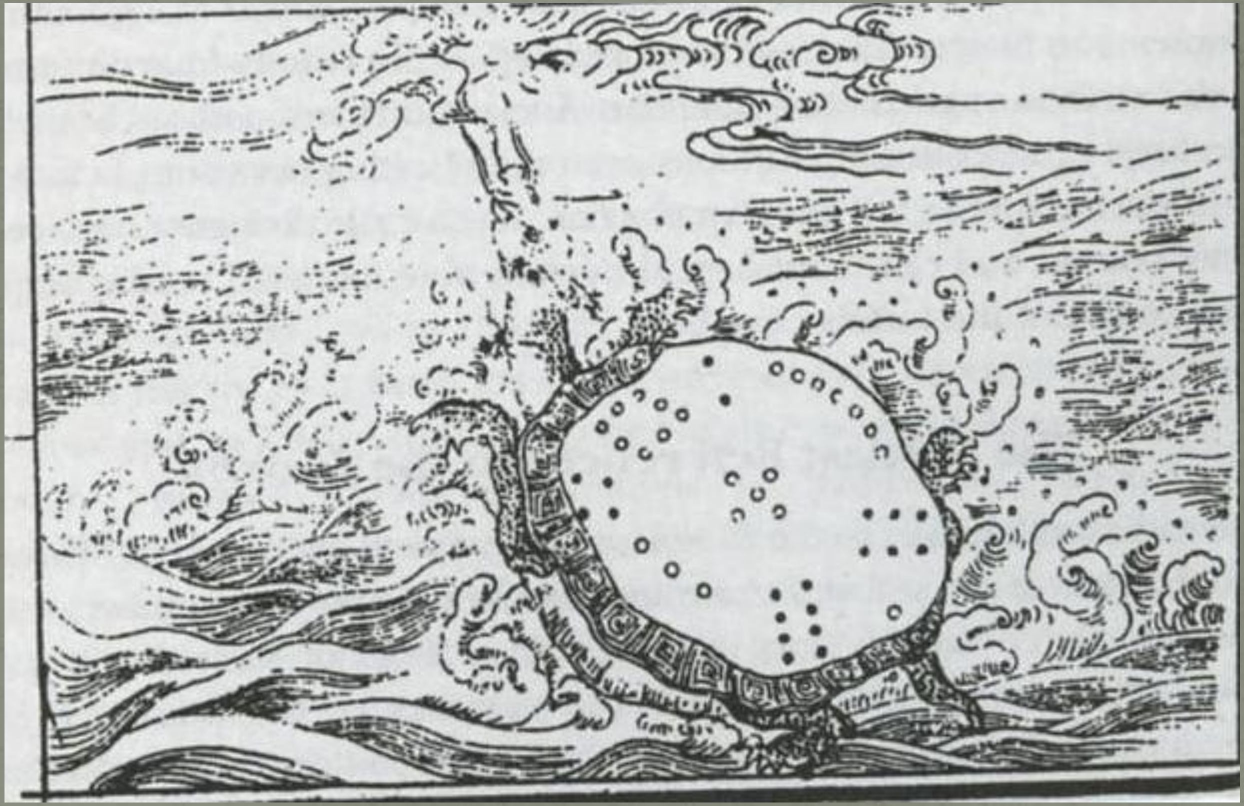
Matteo Ricci

Matteo Ricci

Xu Guangqi

One notable mathematical contribution of Matteo Ricci was the translation of the first six books of Euclid's *Elements* into Chinese with the help of **Xu Guangqi** (1562-1633)

Indigenous Chinese Math: Lo-Shu



Lo-Shu

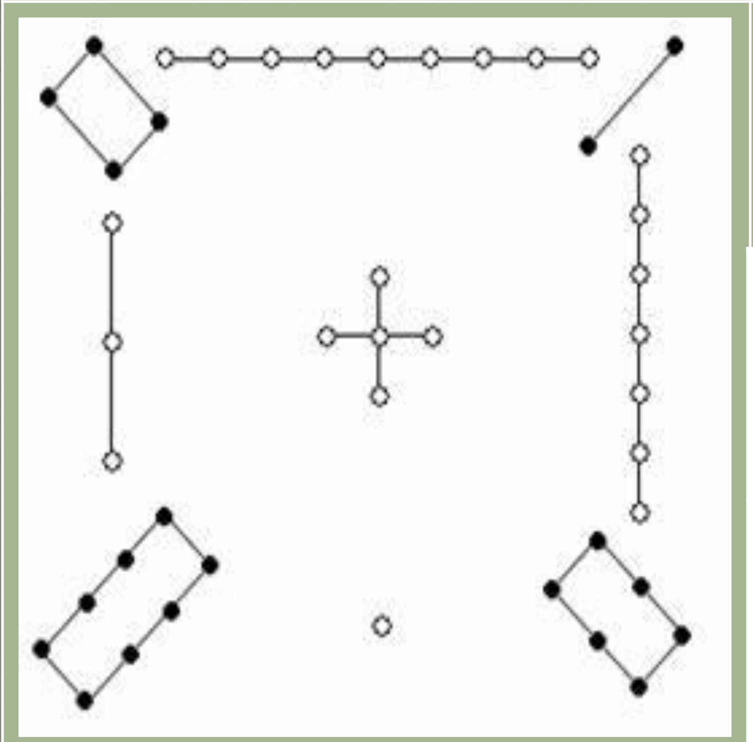
Evidence of Lo-Shu appears between fifth through third century B.C.E.

The tortoise is associated with strength, endurance and immortality.

Emblem of harmony, basis for feng shui.

The Chinese believed that all math and science could be derived from Lo-Shu.

Magic Squares



4	9	2
3	5	7
8	1	6

Estimate for Pi

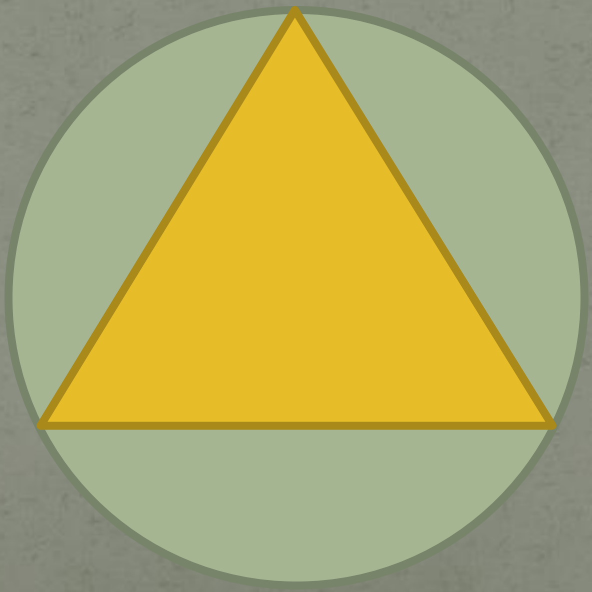
Archimedes (287-212 BCE) used inscribed and circumscribed polygons on a unit circle to estimate Pi.

Archimedes began with a unit circle.

Note: The ratio of the circumference to the diameter is Pi. Since the diameter is one, the circumference is Pi.

Archimedes Estimate for Pi

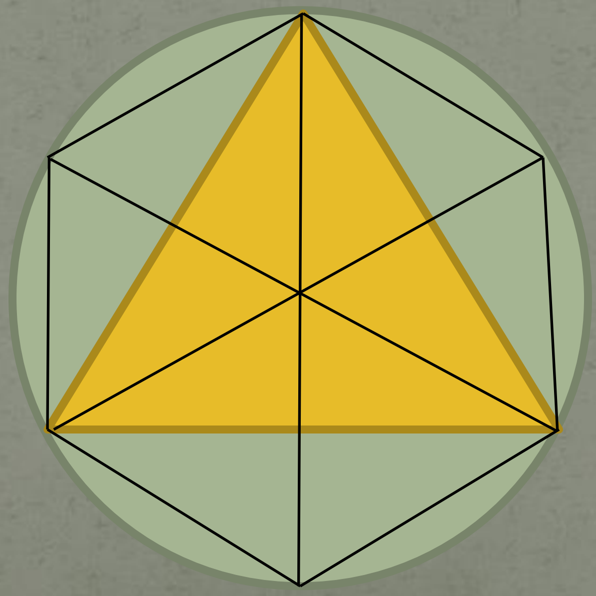
Next he inscribed an equilateral triangle and then determined the perimeter of the triangle.



Archimedes Estimate for Pi

Bisecting each leg of the triangle, Archimedes created a regular hexagon, also inscribed.

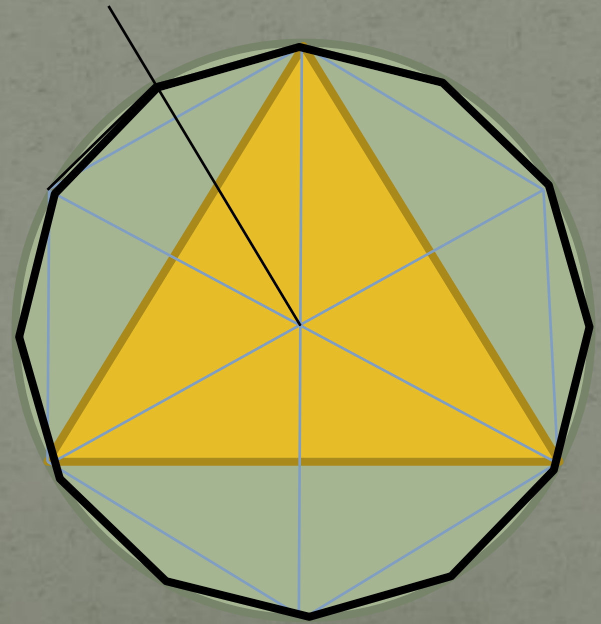
Again, he calculated the perimeter of the regular hexagon.



Archimedes Estimate for Pi

Repeating the process, he bisects the leg of each hexagon and creates a regular 12-gon or dodecagon.

Again he calculates the perimeter.



Archimedes Estimate for Pi

Repeating the process for 24-gon, 48-gon and finally a 96- gon, both inscribed and circumscribed, Archimedes found the following estimation for Pi:

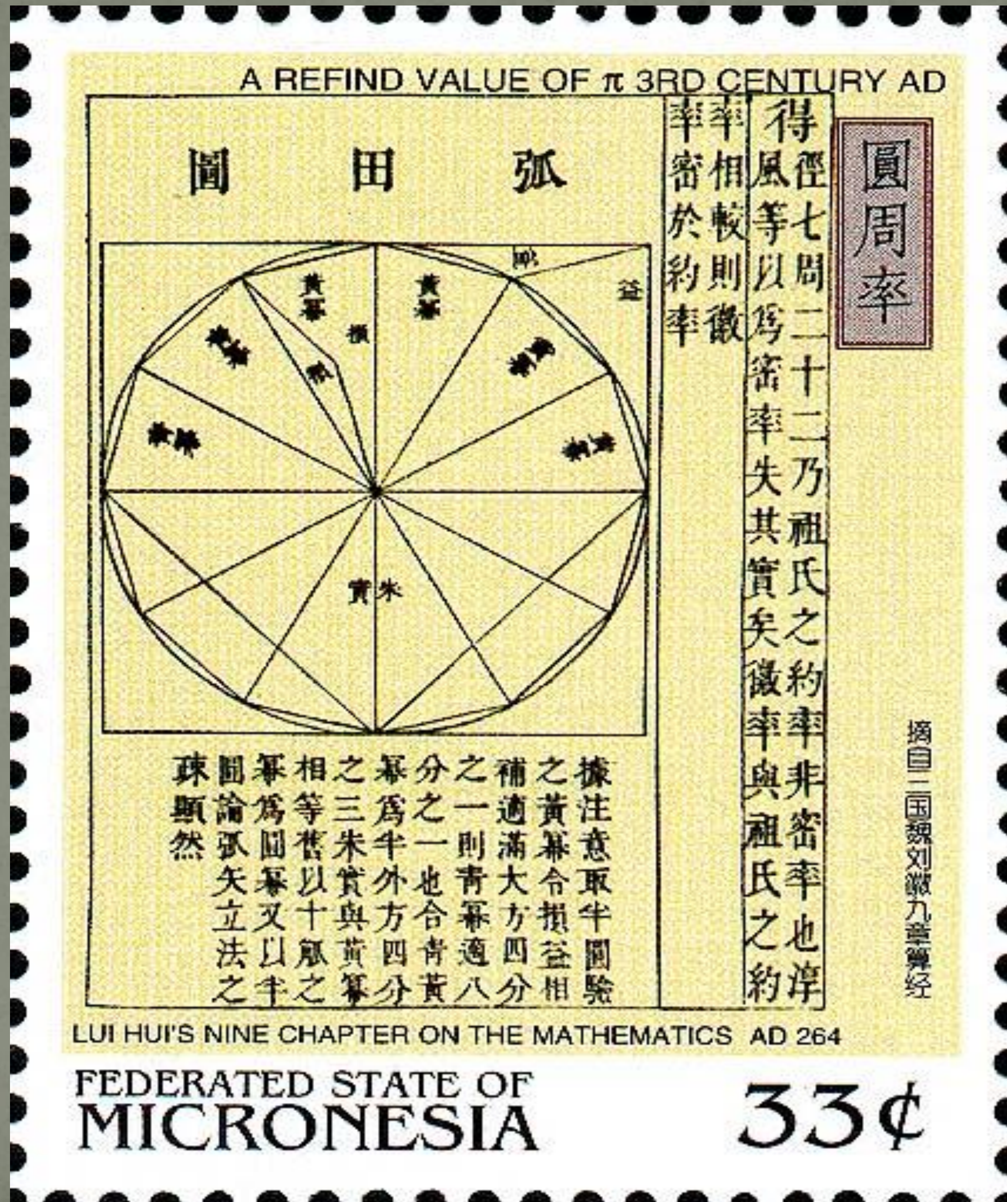
$$3\frac{10}{71} < \pi < 3\frac{1}{7}$$

Liu Hui

Using a method similar to Archimedes, in the third century, Liu Hui calculated the areas of regular polygons with 96 and 192 sides, and approximated pi to be between 3.1410 and 3.1427.

With a polygon of 3,072 sides he determined 3.14159 to be the value of Pi.

Lui Hui


A REFIND VALUE OF π 3RD CENTURY AD
圓周率
弧田圖
LUI HUI'S NINE CHAPTER ON THE MATHEMATICS AD 264
FEDERATED STATE OF
MICRONESIA
33¢

Tsu Ch'ung-Chih

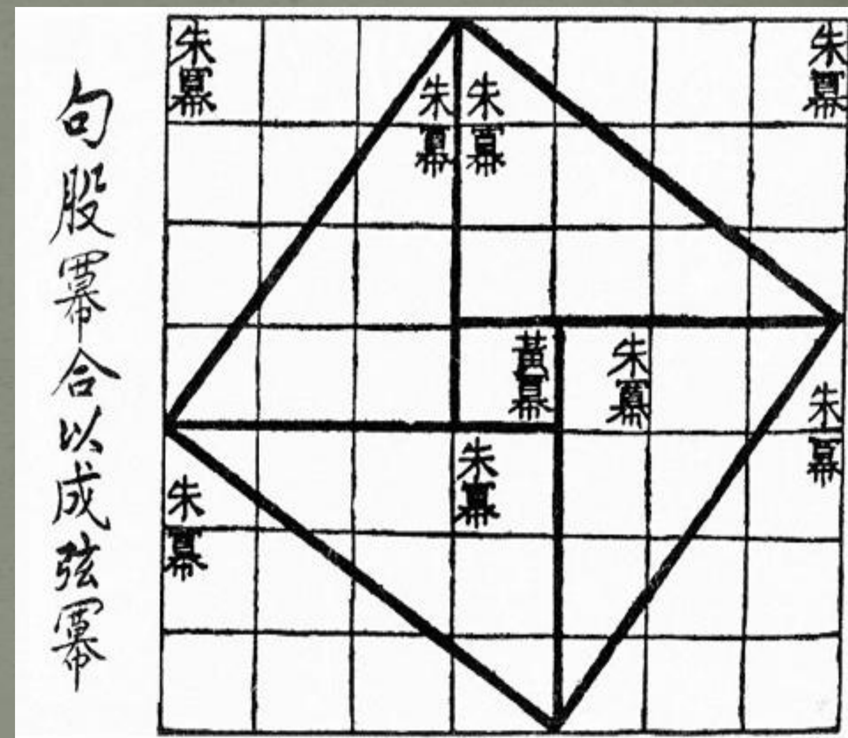
Using regular polygons with 12,288 and 24,576 sides , he calculated pi to be between 3.1415926 and 3.1415927, an accuracy not achieved in the west for another 1000 years.

He also gave the "best" rational approximation, 355/113 , with a three digit denominator, for pi.

Pythagorean Theorem

The Chinese text, “Chou Pei Suan Ching (500-200 BC)” provides a graphical proof of what we come to know as Pythagorean theorem in 200 BCE.

This work was known to Lui Hui as the Gougu Theorem.



Pythagorean Theorem

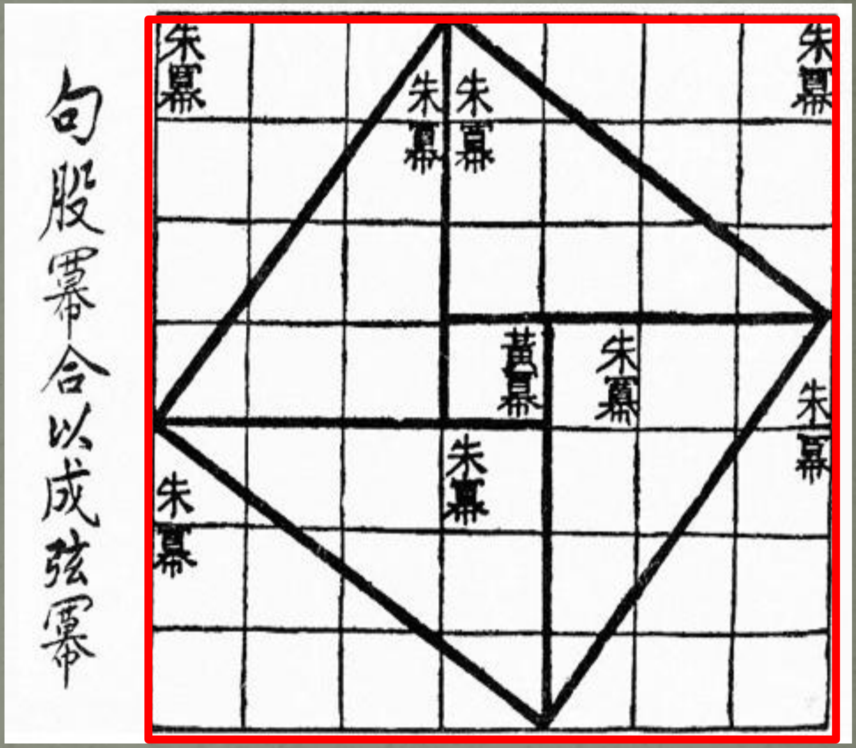
The outside square is 7 x 7.

Consider that 7 = 3 + 4.

The area of the square then is $(3+4)^2$.

Using the binomial expansion:

$(3+4)^2 = 3^2 + 2*3*4 + 4^2$



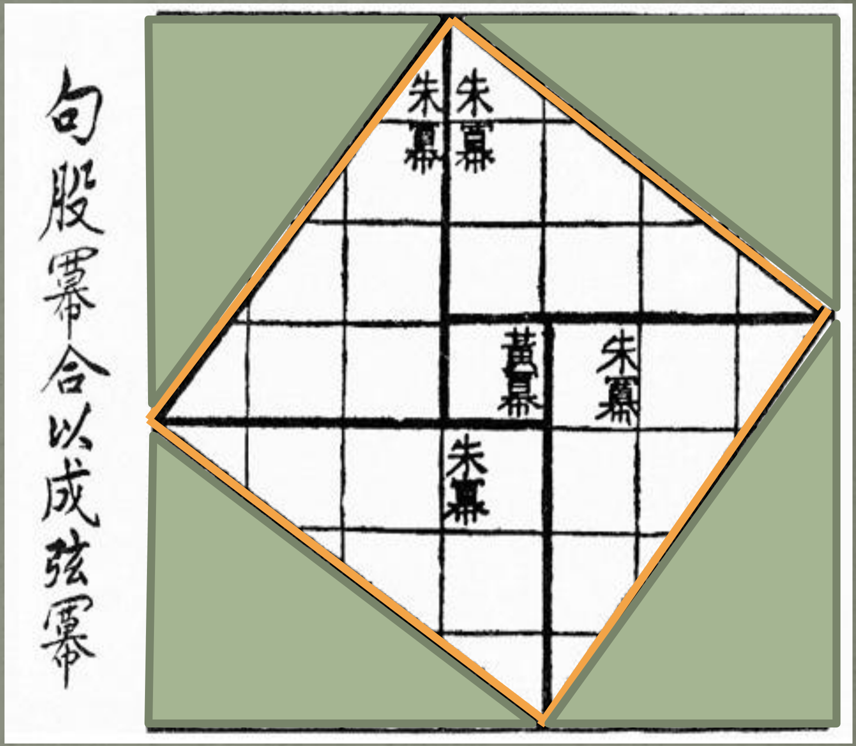
Pythagorean Theorem

To calculate the area of the inner square, we can remove the four surrounding right triangles.

Height = 4, width = 3

Total area of triangles:

(0.5)(4)(3)*4= 2*3*4



Pythagorean Theorem

Area of inner square

= Area of outer Square

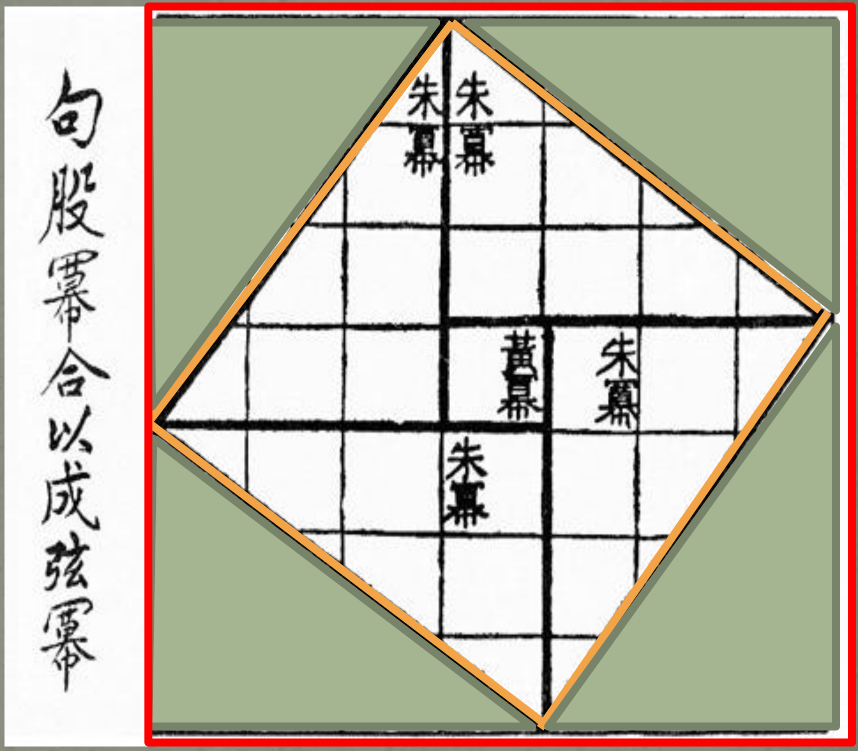
– area of all triangles

$(3+4)^2 - 4*(1/2)*3*4 =$

$3^2 + 2*3*4 + 4^2 - 2*3*4 =$

$3^2 + 4^2 = 25$

$3^2 + 4^2 = 5^2$



Chinese Remainder Theorem

Sunzi suanjing (Master Sun's Mathematical Manual) contained the following problem:

> We have things of which we do not know the number; if we count them by threes, the remainder is 2; if we count them by fives; the remainder is 3; if we count them by sevens, the remainder is 2. How many things are there? (Katz, p. 186)

Chinese Remainder Theorem

Using modern notation:

if we count them by threes, the remainder is 2

$N \equiv 2 \pmod{3}$ $N = 3x + 2$

if we count them by fives; the remainder is 3

$N \equiv 3 \pmod{5}$ $N = 5y + 3$

if we count them by sevens, the remainder is 2

$N \equiv 2 \pmod{7}$ $N = 7z + 2$

Each line is in the form $a \equiv b_i \pmod{m_i}$

Chinese Remainder Theorem

The *Manual* gives the answer as 23. The method used states:

> If you count by threes and have the remainder 2, put 140. If you count by fives and have the remainder 3 put 63. If you count by sevens and the remainder 2 put 30. Add these numbers and you get 233. From this subtract 210 and you get 23 (Katz, p. 186).

The method in the *Manual* notes:

$$70 \equiv 1 (\text{mod } 3) \equiv 0 (\text{mod } 5) \equiv 0 (\text{mod } 7)$$

$$21 \equiv 0 (\text{mod } 3) \equiv 1 (\text{mod } 5) \equiv 0 (\text{mod } 7)$$

$$15 \equiv 0 (\text{mod } 3) \equiv 0 (\text{mod } 5) \equiv 1 (\text{mod } 7)$$

Chinese Remainder Theorem

$a \equiv b_i \pmod{m_i}$

$N \equiv 2 \pmod 3$ $N \equiv 3 \pmod 5$ $N \equiv 2 \pmod 7$

b_i	m_i	$\dfrac{\prod_{i=1}^{n} m_i}{m_i}$	
2	3	$\dfrac{3*5*7}{3} = 35$	$35x \equiv 1 \pmod 3$
3	5	$\dfrac{3*5*7}{5} = 21$	$21x \equiv 1 \pmod 5$
2	7	$\dfrac{3*5*7}{7} = 15$	$15x \equiv 1 \pmod 7$

Chinese Remainder Theorem

$a \equiv b_i \pmod{m_i}$

$N \equiv 2 \pmod{3}$ $N \equiv 3 \pmod{5}$ $N \equiv 2 \pmod{7}$

b_i	$\frac{\prod_{i=1}^{n} m_i}{m_i}$	x	**Product across row**
2	$\frac{3*5*7}{3} = 35$	$x \equiv 2$	2*35*2=140
3	$\frac{3*5*7}{5} = 21$	$x \equiv 1$	3*21*1=63
2	$\frac{3*5*7}{7} = 15$	$x \equiv 1$	2*15*1 =30

Chinese Remainder Theorem

140 + 60 + 30 = 223

Well

$3*5*7 = 105 \equiv 0 \pmod{3} \equiv 0 \pmod{5} \equiv 0 \pmod{7}$

So we can subtract 105 until we get the smallest possible value: 233 – 105 – 105 = 23.

This answer is easy to check.

Problem of the 'hundred fowls"

Two centuries later the Chinese text *Zhang Quijian suanjing* (*Zhang Quijan's Mathematical Manual*) has the first appearance of the "hundred fowls" problem.

"A rooster is worth 5 coins, a hen 3 coins, and 3 chicks 1 coin. With 100 coins we buy 100 of the fowls. How many roosters, hens and chicks are there? (Katz, 187)"

Similar versions of this problem would appear in texts in India, the Islamic world and Europe.

Problem of the 'hundred fowls"

"A rooster is worth 5 coins, a hen 3 coins, and 3 chicks 1 coin. With 100 coins we buy 100 of the fowls. How many roosters, hens and chicks are there? (Katz, 187)"

To solve:

Let *x* represent the number of roosters, *y* the number of hens and *z* the number of chicks.

$$5x + 3y + \frac{1}{3}z = 100$$

$$x + y + z = 100$$

Problem of the 'hundred fowls''

To begin:

$$5x + 3y + \frac{1}{3}z = 100$$

$$x + y + z = 100$$

Simplifying:

$$15x + 9y + z = 300$$

$$x + y + z = 100$$

Removing *z*

$$14x + 8y = 200$$

$$or$$

$$7x + 4y = 100$$

Problem of the 'hundred fowls"

Removing z $$14x + 8y = 200$$

$$or$$

$$7x + 4y = 100$$

Begin to guess. Choose x. You can then solve for y using the simplified equation, listed above. You can solve for z using either of the original equations.

x (roosters)	*y (hens)*	*z (chicks)*
0	25	75
4	18	78
8	11	81
12	4	84

Problem of the 'hundred fowls"

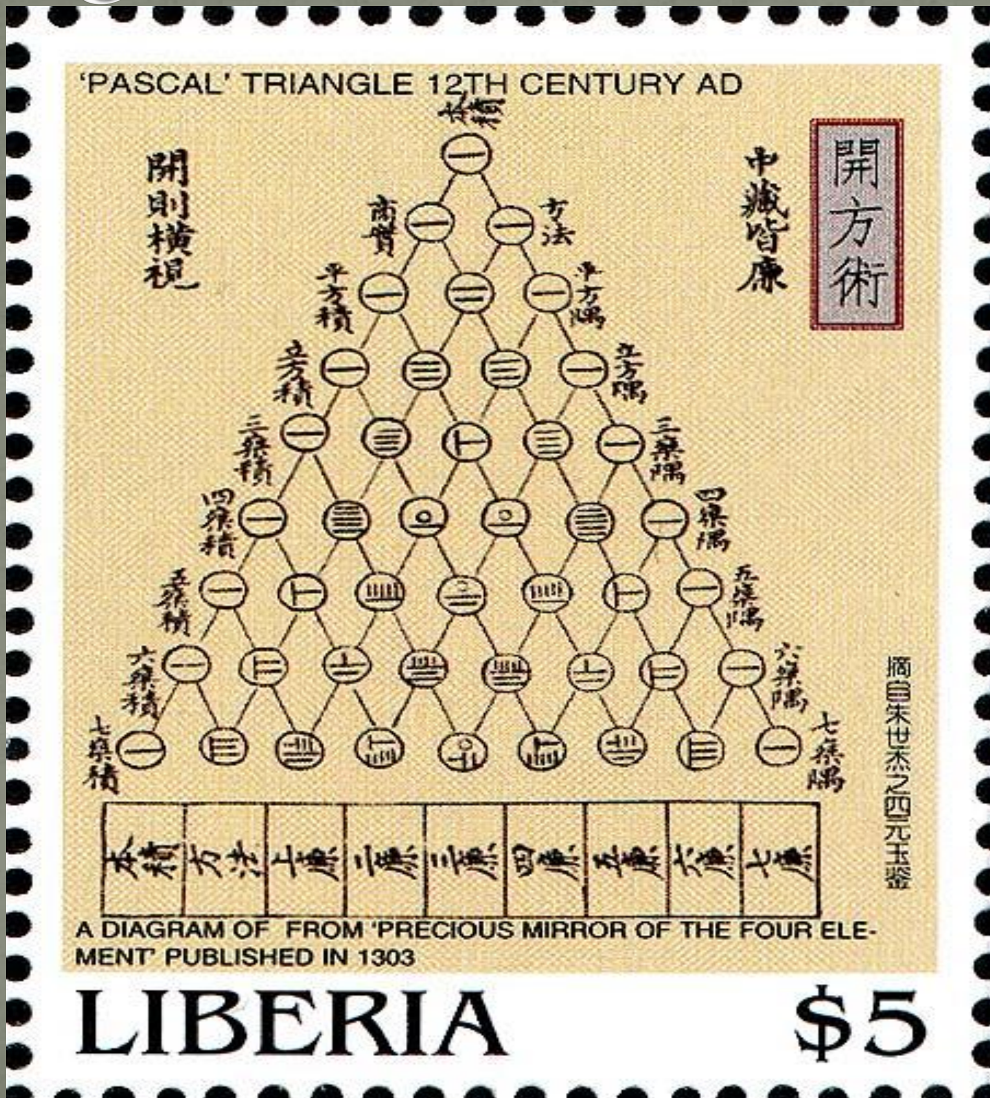
Zhang gives three answers:

4 roosters, 18 hens, 78 chicks
8 roosters, 11 hens, 81 chicks
12 roosters, 4 hens, 84 chicks

To describe his method *Zhang* notes: “Increase the roosters every time by 4, decrease the hens every time by 7 and increase the chicks every time by 3.”

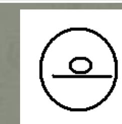
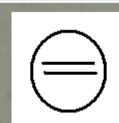
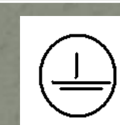
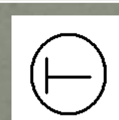
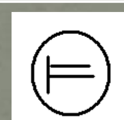
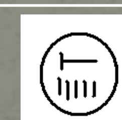
Zhang gives the solutions where all answers are positive.

Indigenous Chinese Math: ?

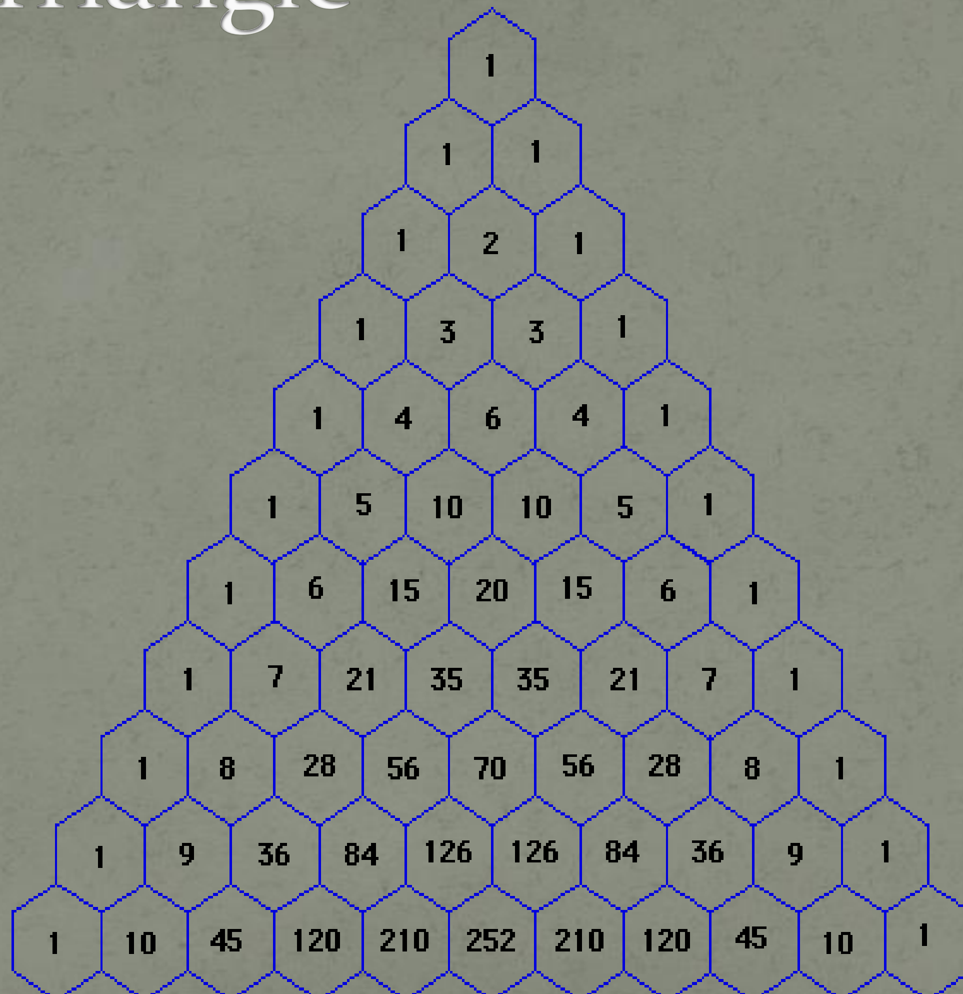


Any guesses?

Chinese Numbering System

1		10	
2		15	
3		20	
4		21	
5		28	
6		35	
7		56	
8		70	

Pascal's Triangle



Binomial Expansion

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

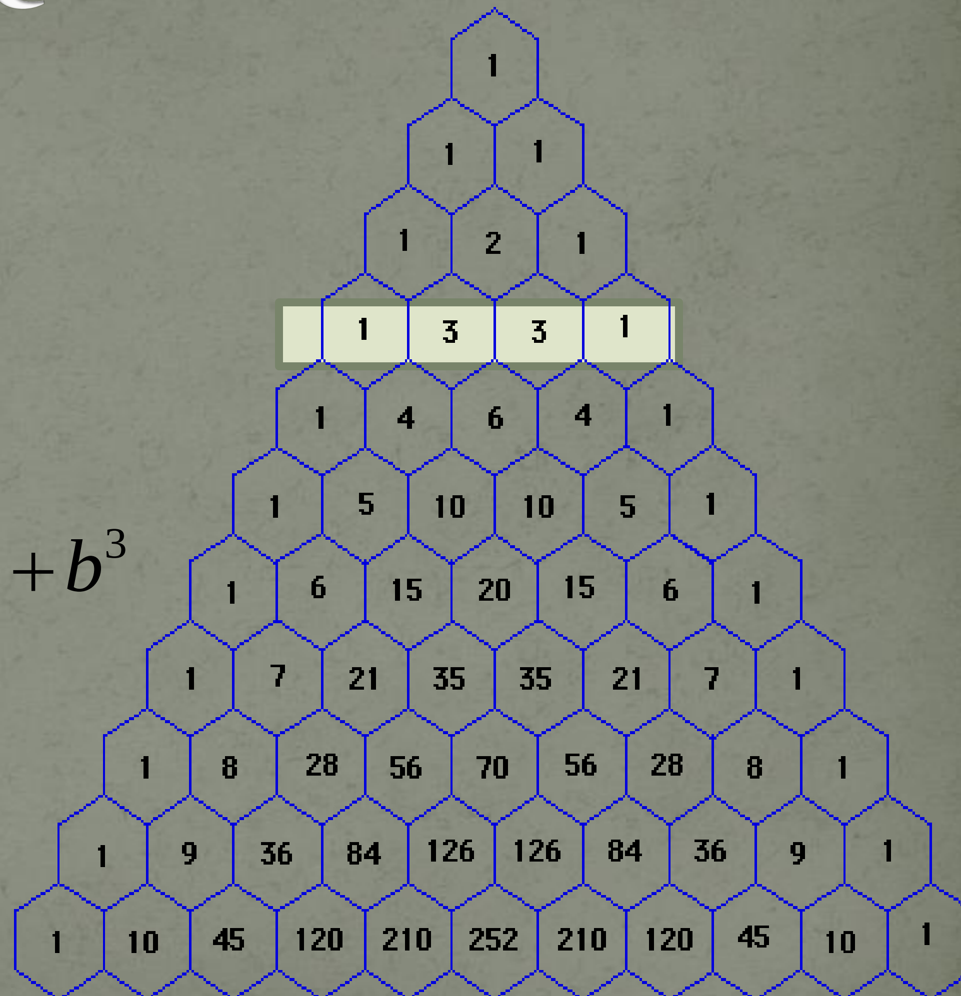
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Pascal's Triangle

Each row makes up the coefficients of the binomial expansion of

$(x + y)^n$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$



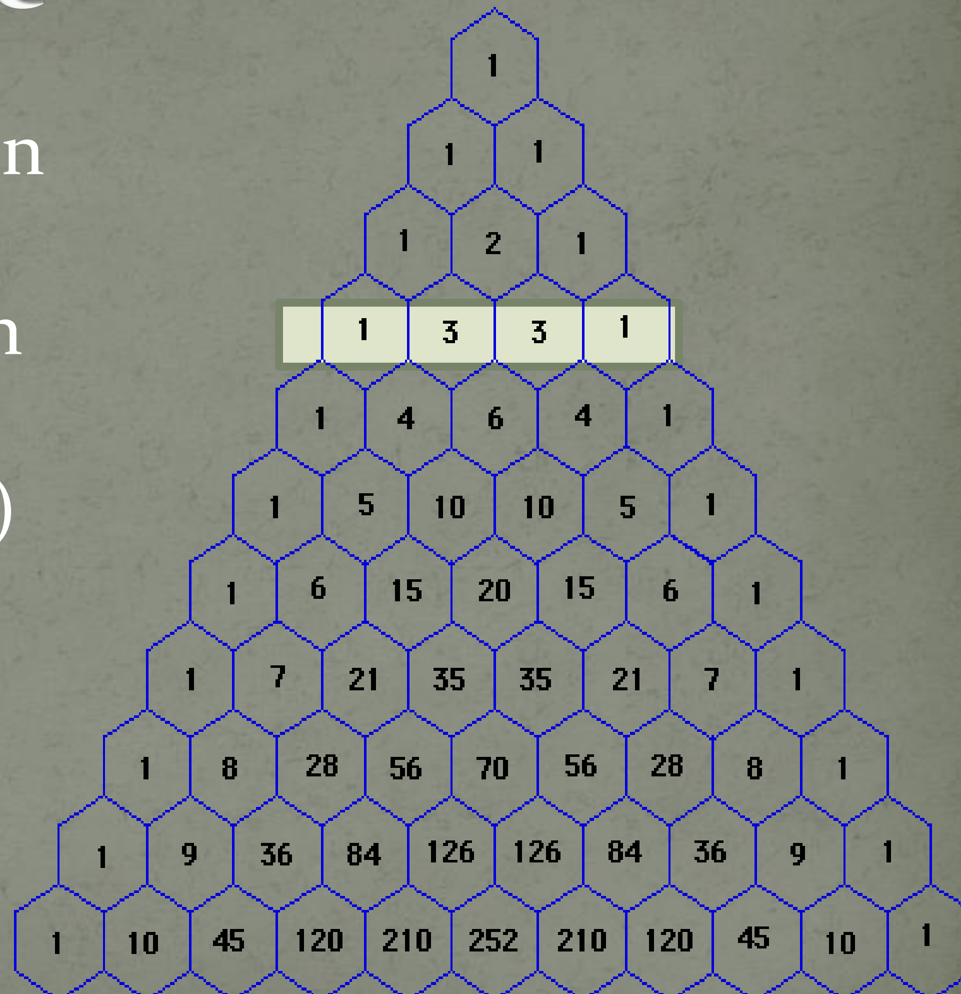
Pascal's Triangle

11^n

Suppose n=3, then 11^3 = 1331

Pascal's Triangle

The sum of the numbers in any row is equal to 2 to the n^{th} power or 2^n, when n is the number of the row. (Starting with n= 0)



Suppose n=3, then $2^3 = 8$

1+3+3+1 = 8

Pascal's Triangle

Combinatorics: Suppose you have *n* objects. How many ways can I choose *r* of them if order is not important?

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

Consider the example where *n* = 3 and *r* = 0, 1, 2, 3.

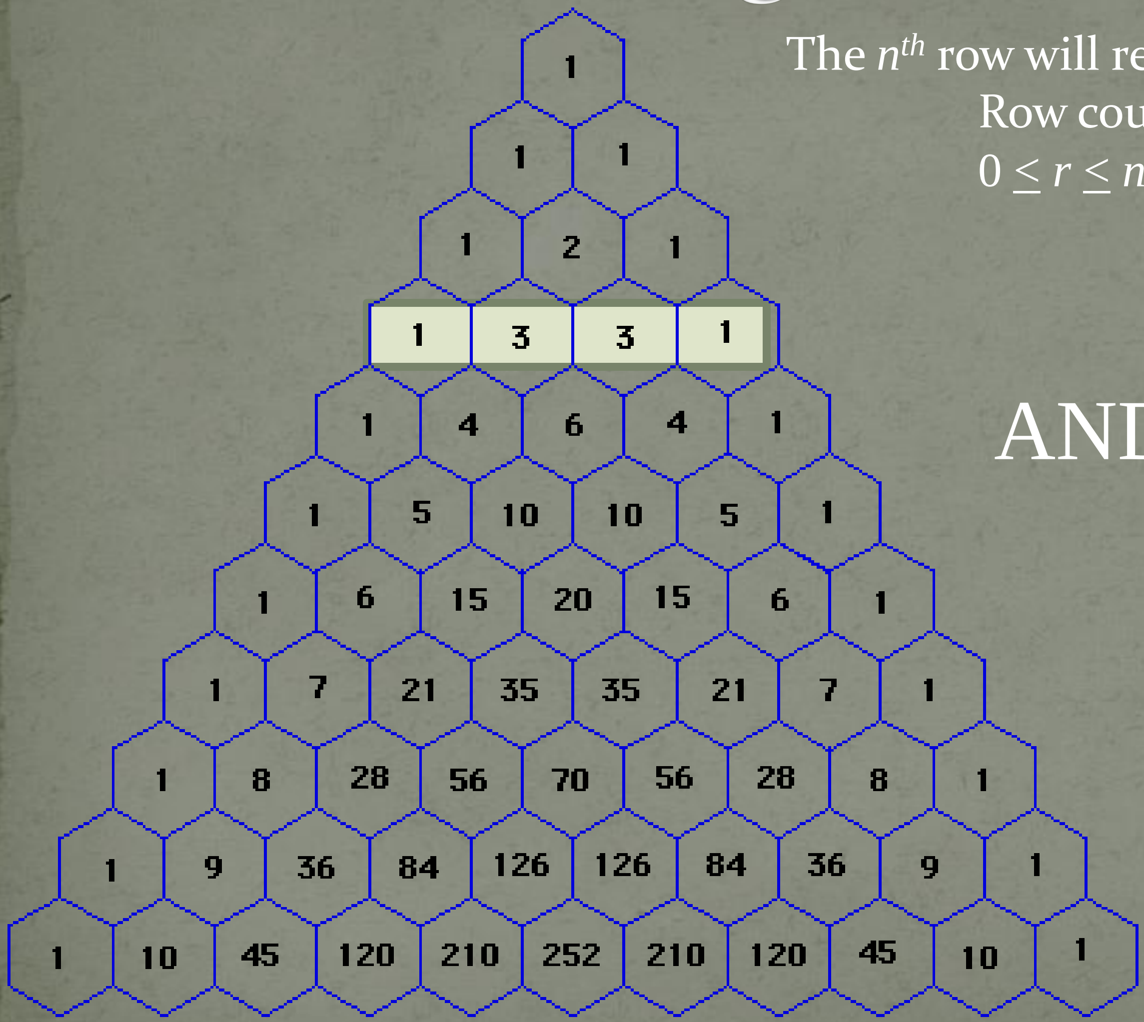
$$C(3,0)=\frac{3!}{0!(3-0)!}=1$$

$$C(3,1)=\frac{3!}{1!(3-1)!}=3$$

$$C(3,2)=\frac{3!}{2!(3-2)!}=3$$

$$C(3,3)=\frac{3!}{3!(3-3)!}=1$$

Pascal's Triangle



The n^{th} row will return the value of $C(n,r)$

Row counting starts at $n = 0$

$0 \leq r \leq n$

$$\text{AND: } \sum_{r=0}^{n} C(n,r) = 2^n$$

Pascal's Triangle

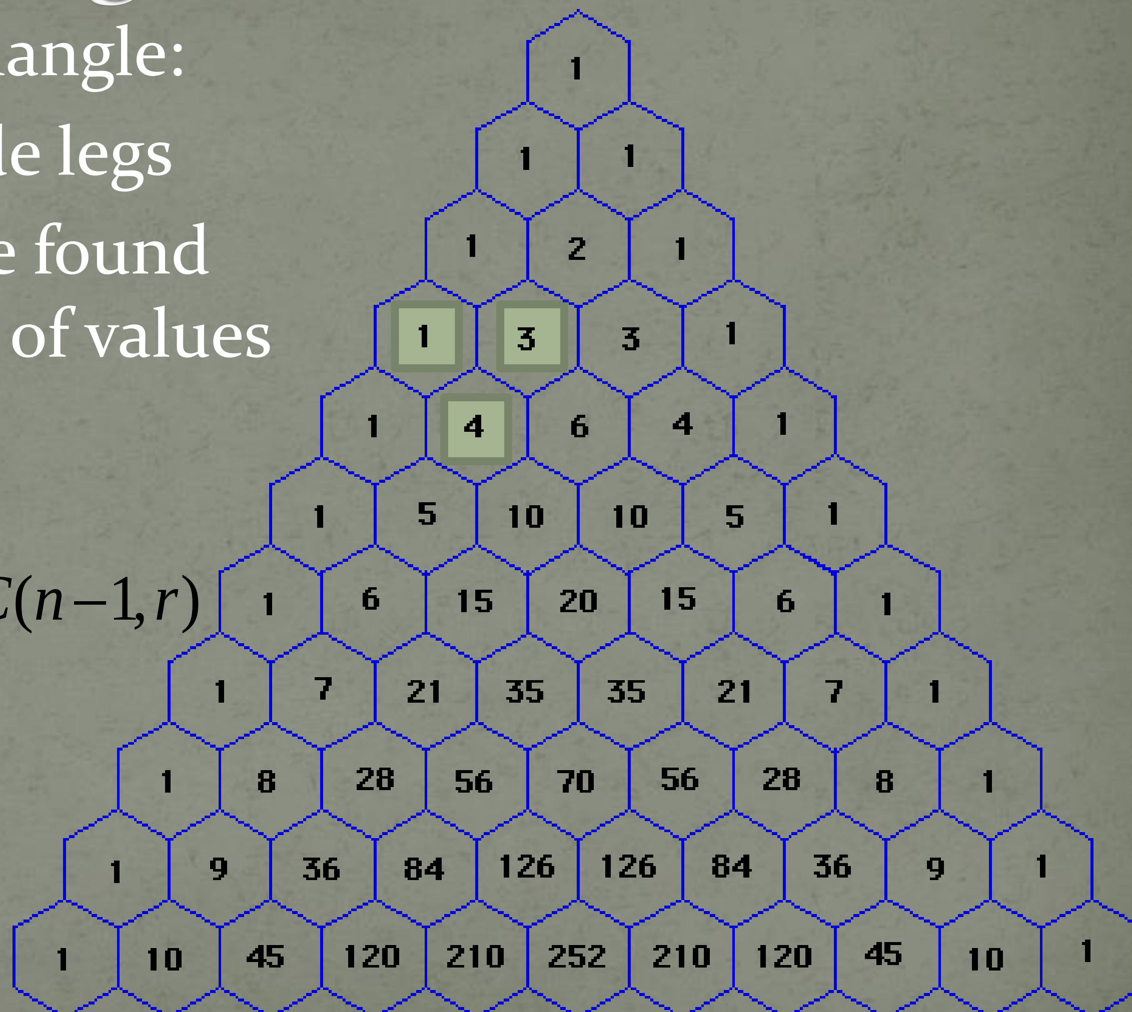
To generating the triangle:

Keep 1 on the outside legs

The inside values are found by adding the pair of values preceding it.

Thus:

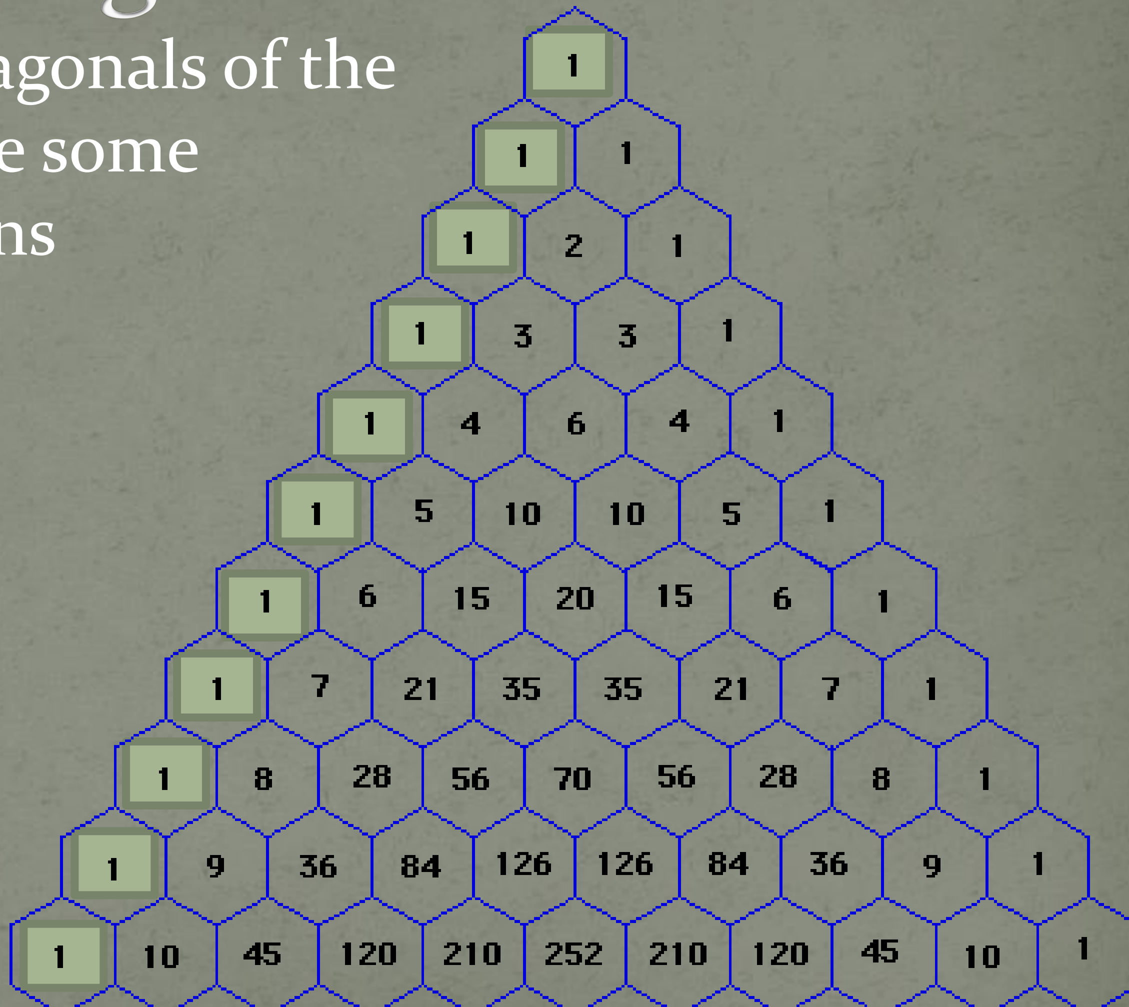
$$C(n,r) = C(n-1,r-1) + C(n-1,r)$$



Pascal's Triangle

If we look on the diagonals of the triangle we can see some interesting patterns

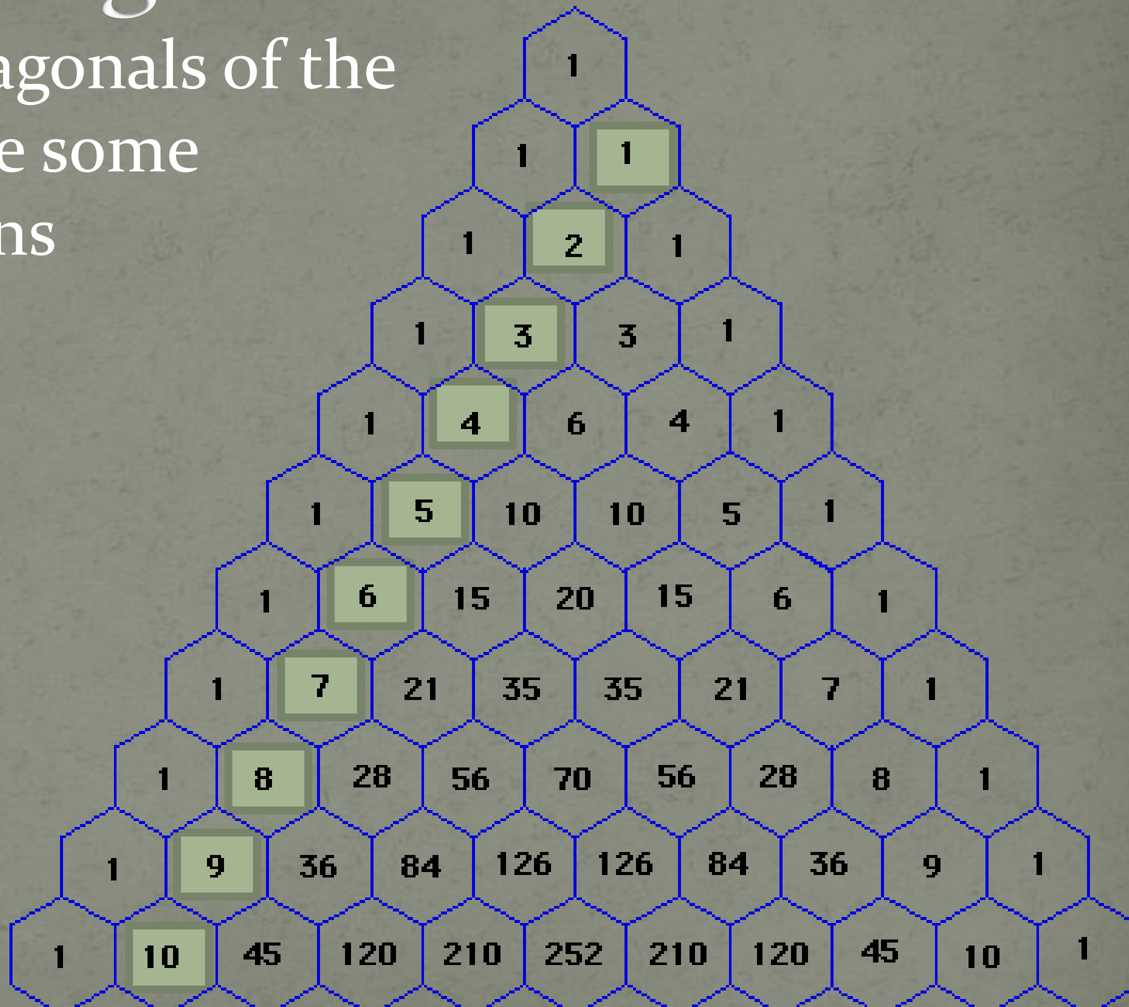
Ones



Pascal's Triangle

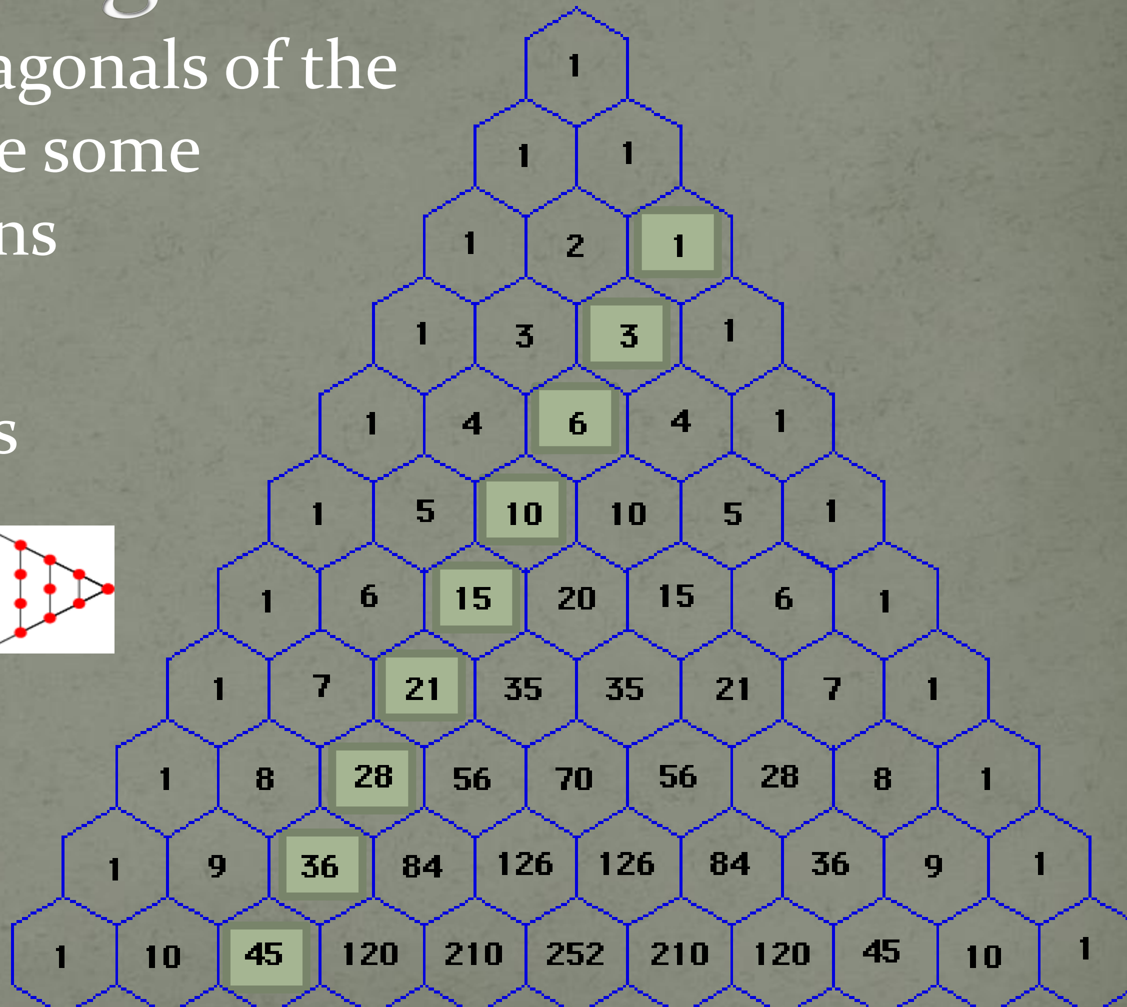
If we look on the diagonals of the triangle we can see some interesting patterns

Counting Numbers

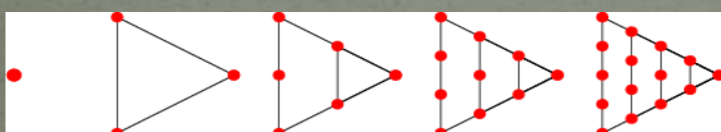


Pascal's Triangle

If we look on the diagonals of the triangle we can see some interesting patterns



Triangular Numbers

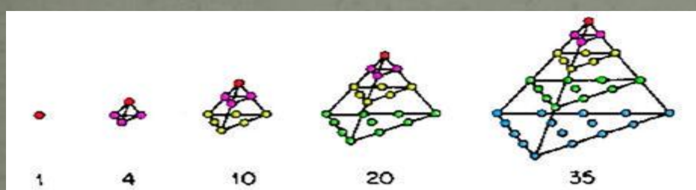


$$\sum_{i=1}^{n} i$$

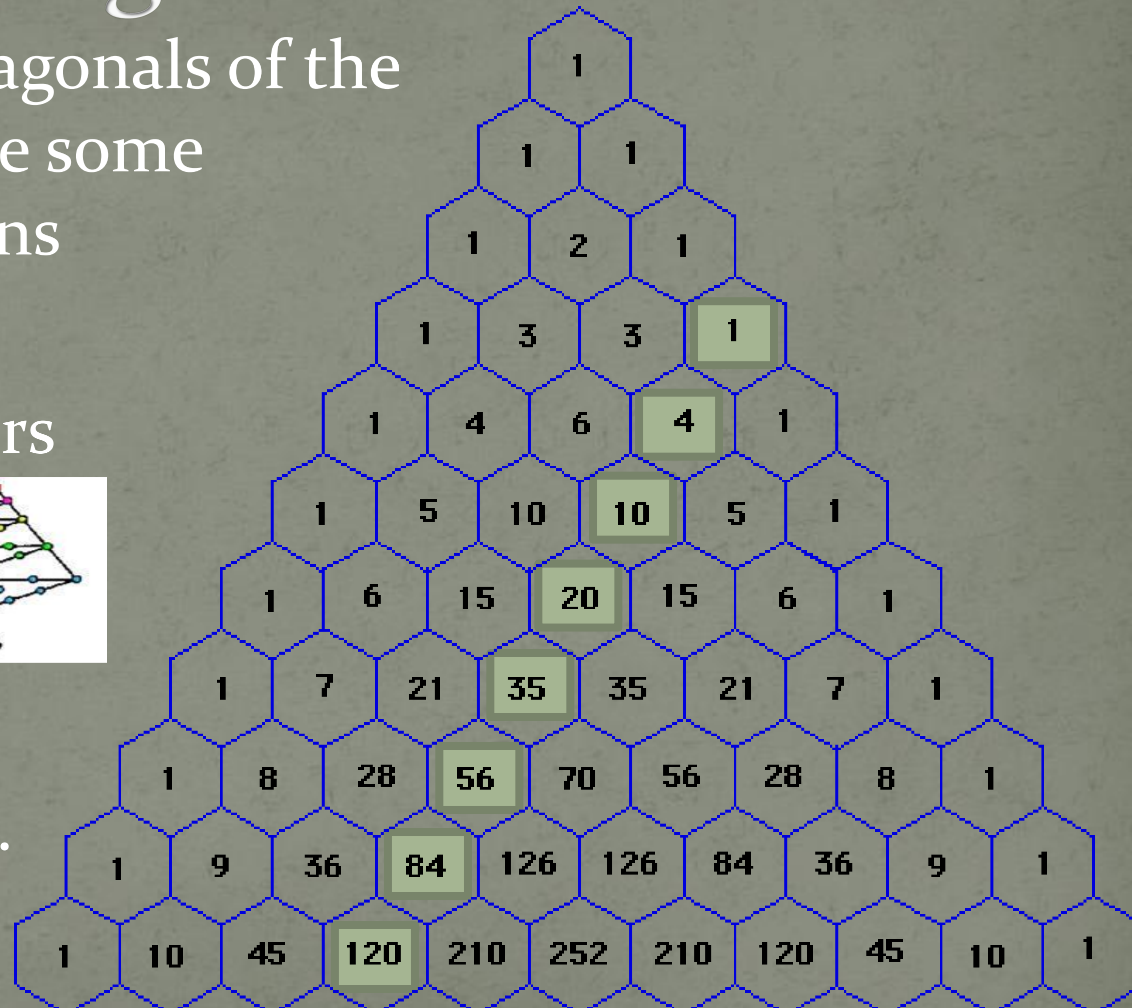
Pascal's Triangle

If we look on the diagonals of the triangle we can see some interesting patterns

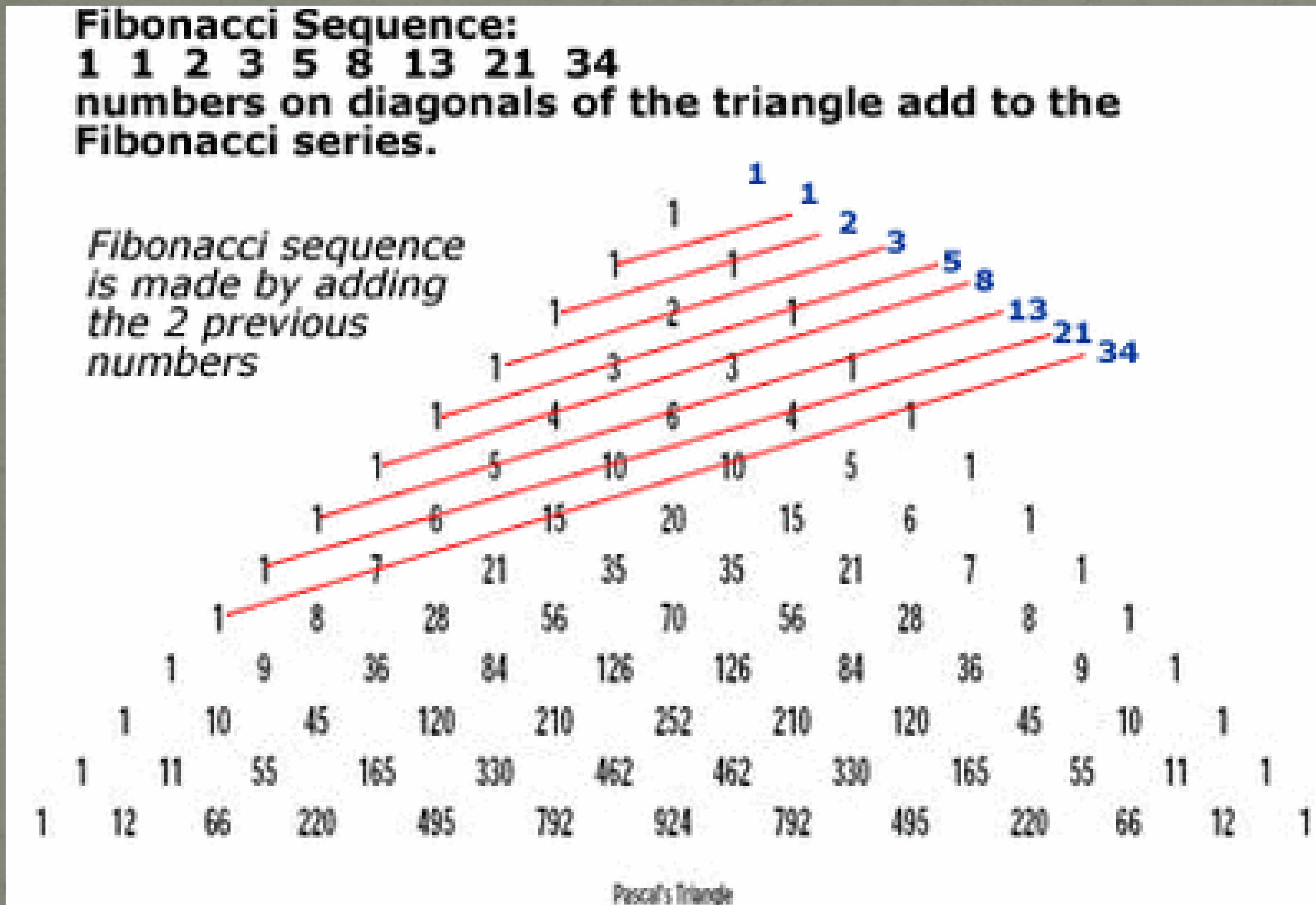
Tetrahedral Numbers



Sum of the triangular numbers.



Pascal's Triangle



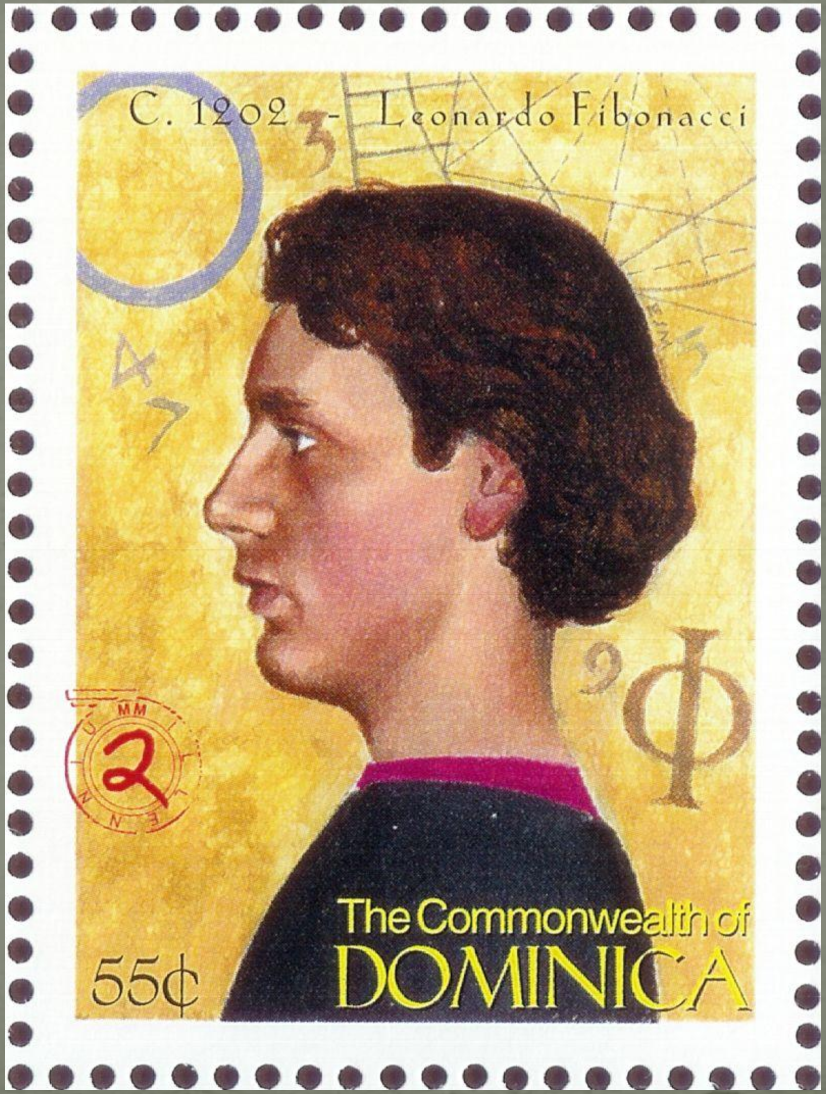
Blaise Pascal

Evidence substantiates that the Chinese knew about Pascal's Triangle as early as 1261.

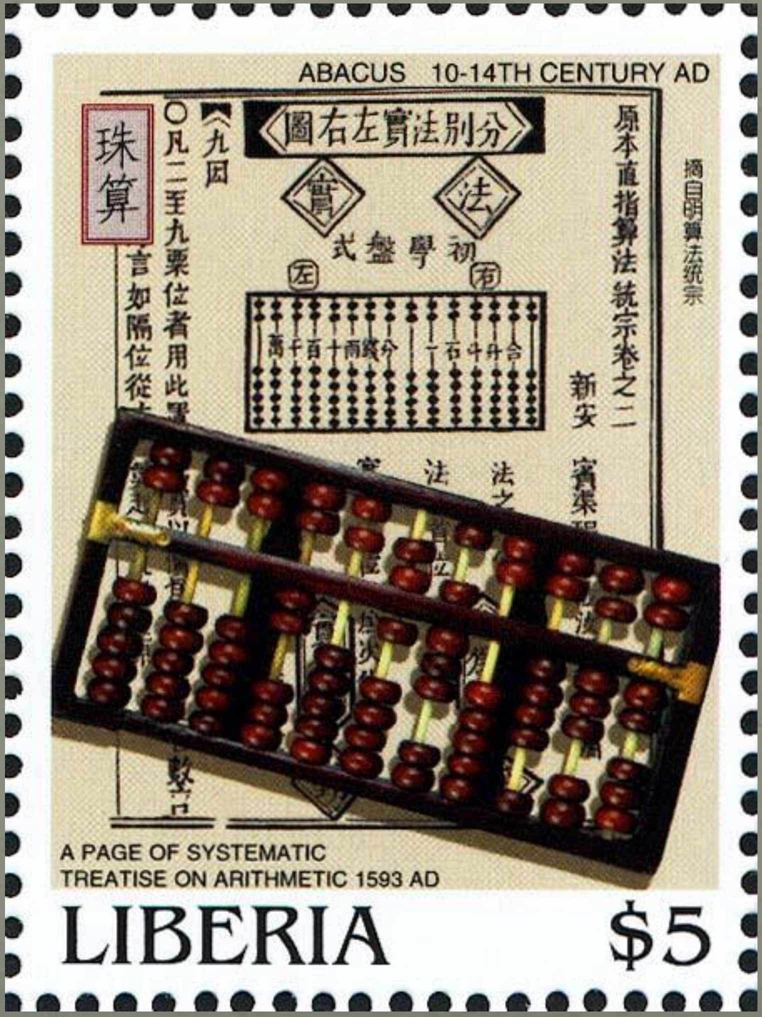
Blaise Pascal received credit for the triangle in 1653, with the publication of his text *Traité du triangle arithmétique* (Treatise on the Arithmetical Triangle).

Leonardo of Pisa, aka Fibonnaci

Leonardo of Pisa or **Fibonacci published** *Liber abaci* in 1202.



Abacus


ABACUS 10-14TH CENTURY AD
珠算
A PAGE OF SYSTEMATIC
TREATISE ON ARITHMETIC 1593 AD
LIBERIA
$5

Problem of the 'hundred fowls"

"A rooster is worth 5 coins, a hen 3 coins, and 3 chicks 1 coin. With 100 coins we buy 100 of the fowls. How many roosters, hens and chicks are there? (Katz, 187)"

To solve:

Let x represent the number of roosters, y the number of hens and z the number of chicks.

$$5x + 3y + \frac{1}{3}z = 100$$

$$x + y + z = 100$$

Problem of the 'hundred fowls"

Given $$5x+3y+\frac{1}{3}z=100$$

$$x+y+z=100$$

Using Gaussian elimination:

$$\left[\begin{array}{ccc|c} 5 & 3 & \frac{1}{3} & 100 \\ 1 & 1 & 1 & 100 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & \frac{3}{5} & \frac{1}{15} & 20 \\ 0 & 1 & \frac{7}{3} & 200 \end{array}\right]$$

We can see from the matrix we do not have a unique solution.

Problem of the 'hundred fowls"

We can see from the matrix that both the values of x and y depend on z. With some simplification we see:

$$y = \frac{-7}{3} z + 200$$

$$x = \frac{4}{3} z - 100$$

The parametric solution:

$$z = 3t$$

$$y = -7t + 200$$

$$x = 4t - 100$$

Another example:

You have *c* pennies, *d* dimes, *q* quarters, 100 coins in all. The coins add up to $5.00. How many pennies, dimes and quarters do you have?

$$c + d + q = 100$$

$$0.01c + 0.10d + 0.25q = \$5.00$$

Another example:

Simplifying:

$$c + d + q = 100$$

$$c + 10d + 25q = 500$$

Removing *c*

$$9d + 24q = 400$$

Another example:

$$9d + 24q = 400$$

d (dimes)	*q (quarters)*	*c (pennies)*

False Position

In *Jiuzhang Suanshu (The Nine Chapters of Mathematical Art*) Chapter 7 presents to the reader a method for solving linear equations.

The method is known as False position and is referred to by *Juizhang* as Surplus and Deficiency.

False Position

Problem:

Now chickens are purchased jointly; everyone contributes 9, the excess is 11; everyone contributes 6, the deficit is 16. Tell the number of people, the item price, what is each (Kangshen, et. al p. 358)?

Method for solution:

To relate the excess and deficit for the articles jointly purchased: lay down the contribution rates. Subtract the smaller from the greater, take the remainder to reduce the divisor and the dividend. The [reduced] dividend is the price of an item. The [reduced] divisor is the number of people (Kangshen, et. al., p. 359).

Linear Thinking

Rates	Excess or deficit
9	11
6	16

The difference in the rates is 3.

Cross multiply and add:

9*16 + 6*11 = 210;

210/3 = 70, the price

11+16=27

27/3 = 9, the number of people.

False Position

The error amount for this example, was first an excess and then a deficit. In the case that either both were deficit or excess, we would look at the difference rather than the sum of the errors.

False Position

The Jiuzhang was the dominant mathematical text in China until 1600 AD.

Evidence of using the method of False Position to solve linear equations can be found in several ancient cultures, including Babylonia, India, and Egyptian.

Problem Solving

Zhang Qiujian's Mathematical Manual states:

Each pint of high quality wine costs 7 coins, each pint of ordinary wine costs 3 coins and 3 pints of wine dregs cost 1 coin. If 10 coins are used to buy 10 pints in all, find the amount of each type and the total money spent on each.

x represents high quality wine, y represent ordinary wine, z represent wine dregs.

$$7x + 3y + \frac{1}{3}z = 10$$

$$x + y + z = 10$$

Problem Solving

Simplifying:

$$21x + 9y + z = 30$$

$$x + y + z = 10$$

Removing *z*:

$$20x + 8y = 20$$

$$or$$

$$5x + 2y = 5$$

Problem Solving

Guessing: $5x + 2y = 5$

x (high quality)	*y (ordinary)*	*z (dregs)*
1	0	9

Work Cited

Texts:

Beckmann, Petr. *A History of Pi.* New York: St. Martin's Press, 1971.

Dudley , Underwood. *Elementary Number Theory,* Dover Publication, 2008.

Ifrah, Georges. *From One to Zero: A Universal History of Numbers.* New York: Viking, Penguin, 1985.

Kangshen Shen, Crossley John, N., Lun Anthony W. -C. The Nine Chaptrs on the Mathematical Art Companion and Commentary. © 1999, Oxford University Press.

Katz, Victor. *A History of Mathematics An Introduction.* © 1993. Harper Collins College Publishers

Work Cited

Ore, Oystein. *Number Theory and its History.* New York: McGraw-Hill Inc., 1948.

Spence, Jonathan. *The Memory Palace of Matteo Ricci.* New York: Viking Penguin, 1984.

Swetz, Frank J., with Kao, T. I *Was Pythagoras Chinese? An Examination of Right Triangle Theory in Ancient China*. University Park, PA: The Pennsylvania State University Press, 1977.

Swetz, Frank J., ed. *From Five Fingers to Infinity: A Journey through the History of Mathematics.* Chicago: Open Court, 1994.

Swetz, Frank J., ed. *Legacy of LuoShu.* Chicago: Open Court, 2004.

Swetz, Frank, et al., eds. *Learn from the Masters.* Washington, DC: Mathematical Association of America, 1995.

Work Cited

Images and Websites:

http://milan.milanovic.org/math/english/tetrahedral/tetrahedral.html

http://milan.milanovic.org/math/english/triangular/triangular.html

http://jeff560.tripod.com/stamps.html

http://www.unisanet.unisa.edu.au/07305/pascal.htm

http://math.about.com/od/algebralessons/ss/Pascal_3.htm

http://www.faculty.fairfield.edu/jmac/sj/scientists/ricci.htm